Identification of damage using low frequency harmonics in trusses and beams

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Abstract

The aim of this work is to present a damage identification method dedicated to truss and frame structures. The core of the approach is the Virtual Distortion Method, which is a fast reanalysis method successfully applied to damage identification. Loss of mass and stiffness are modelled by virtual distortions and modifications of the parameters are calculated as a result of a sensitivity-based minimisation. In this paper we deal with a steady-state problem i.e. low frequency, non-resonance harmonic excitation induces a static-like structural response with virtual distortions (design variables) modelling parameter modifications.

1 Introduction

The presented approach to damage identification is a continuation of research done within the PiezoDiagnostics (PD) project [1]. The general purpose of the PD project was identification of corrosion (or damage of considerable extent) in pipelines. Generation and detection of a global structural mode by piezo-actuators and sensors was tested in the PD project. Perturbations of the mode due to various damage scenarios were investigated. A software tool, based on the Virtual Distortion Method (VDM), was developed [2]. The tool is able to perform damage identification via the solution of an inverse, dynamic problem in the time domain thanks to employing gradient-based optimisation. As VDM-based identification belongs to the class of model updating methods, a well-calibrated FE model is required in order to produce meaningful results with experimental data. In this paper, the possibility of carrying out the analogous damage identification in the frequency domain is explored. The principal motivation for developing the new frequency-based approach was the reduction of vast consumption of computational time, observed in the previous approach. In the first step, a simplified dynamic problem with no damping is considered. A number of selected excitation frequencies of low range (below 1 kHz) are the subject of analysis. The proposed approach has been implemented in a software code [3]. Two structural models are used to show the effectiveness of the new software – truss and beam. In both models, stiffness and mass reduction are considered as damage parameters. Experimental verification of the approach using a 3D truss structure is on the way.

2 VDM for a steady-state problem

The use of VDM in dynamic damage identification was previously discussed in [4]-[6]. The considerations concerned modifications of stiffness parameters of truss and frame-like structures in the time domain, in which dynamic analysis using the VDM is numerically time consuming. In this paper an alternative approach to damage identification in the frequency domain is discussed.

2.1 Virtual distortions and modification parameters

The virtual distortion is an initial perturbation introduced to a finite element of an original structure subjected to external excitation realised by the load component $p^0_k(t,\omega)$ – modelling inertia modification or the strain component $\varepsilon^0_\alpha(t,\omega)$ – modelling stiffness modification (t, ω denote time and frequency of excitation, respectively).

For a truss finite element α, the deformation state is determined by just one strain component $\varepsilon_\alpha(t,\omega)$. Relation between virtual distortions $\varepsilon^0_\alpha(t,\omega)$, $p^0_k(t,\omega)$ and the modified stiffness parameter $\mu_\alpha = \frac{\hat{k}^{EA}_\alpha}{k^{EA}_\alpha}$ called *modification parameter*, is expressed by the formula (see [4]):

$$\frac{\hat{k}_{EA}}{k_{EA}}\,\varepsilon_\alpha(t,\omega) = \varepsilon_\alpha(t,\omega) - \varepsilon^0_\alpha(t,\omega) \qquad \text{or} \qquad \mu_\alpha\,\varepsilon_\alpha(t,\omega) = \varepsilon_\alpha(t,\omega) - \varepsilon^0_\alpha(t,\omega), \tag{1}$$

Let us notice, that the modifications of both the Young's modulus E as well as cross-section area A of an element α can be modelled. Moreover, the updated strain $\varepsilon_\alpha(t,\omega)$ depends on virtual distortions $\varepsilon^0_\alpha(t,\omega)$ and $p^0_k(t,\omega)$, thus the Eqn (1) is non-linear.

Any deformation state for a 2D-Beam finite element specifies 3 components (orthogonal base) obtained by solving the eigenvalue problem of its stiffness matrix. On this element, the virtual distortions corresponding to the 3 components are imposed. The virtual distortions have an oscillating form (presented in Fig. (1) for amplitude values) with frequency of excitation ω. The relation between the modification parameters μ_α and

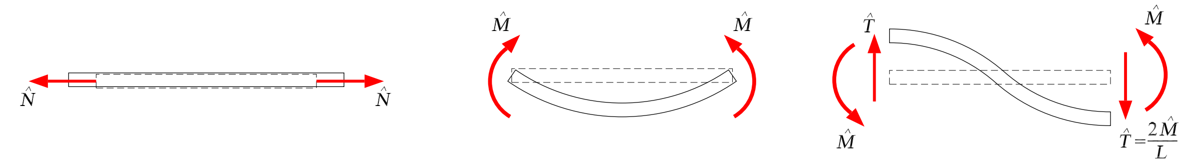

Figure 1: Basic virtual distortion states.

distortions come from the following postulate: *The response of the modelled structure by virtual distortion has to be identical to the modified response in the sense of the strain and stress fields.* For 2D-Beam finite element, the relations analogous to Eqn (1) are expressed by the equations:

$$\begin{gathered} \mu^{(1)}_\alpha\,\varepsilon^{(e)}_\alpha(t,\omega) = \varepsilon^{(e)}_\alpha(t,\omega) - \varepsilon^{(e)\,0}_\alpha(t,\omega), \\ \mu^{(2)}_\alpha\,\kappa^{(e)}_\alpha(t,\omega) = \kappa^{(e)}_\alpha(t,\omega) - \kappa^{(e)\,0}_\alpha(t,\omega), \qquad \mu^{(3)}_\alpha\,\chi^{(e)}_\alpha(t,\omega) = \chi^{(e)}_\alpha(t,\omega) - \chi^{(e)\,0}_\alpha(t,\omega). \end{gathered} \tag{2}$$

The first equation of the set (2) concerns axial stiffness (similarly to Eqn (1)) and the remaining ones describe bending states, where $\mu^{(2)}_\alpha = \mu^{(3)}_\alpha = \frac{\hat{k}^{EJ}_\alpha}{k^{EJ}_\alpha}$ is the ratio of a modified bending stiffness to the original one. Further, we assume the modifications of cross-section area ($\mu^{(1)}_\alpha = \frac{\hat{A}_\alpha}{A_\alpha}$) and moment of interia ($\mu^{(2)}_\alpha = \mu^{(3)}_\alpha = \frac{\hat{J}_\alpha}{J_\alpha}$) independently. For the whole structure, the vectors of strains and stiffness modification parameters are built:

$$\varepsilon_\alpha(t,\omega) = \{\varepsilon^{(e)}_1,\, \kappa^{(e)}_1,\, \chi^{(e)}_1, \ldots \varepsilon^{(e)}_n,\, \kappa^{(e)}_n,\, \chi^{(e)}_n\}, \tag{3}$$

$$\mu_\alpha = \{\mu^{(1)}_1, \mu^{(2)}_1, \mu^{(3)}_1, \ldots \mu^{(1)}_n, \mu^{(2)}_n, \mu^{(3)}_n\}. \tag{4}$$

The vector of the virtual distortions have an analogous form to Eqn (3). Thus, the Eqn (2) can be expressed concisely for the whole structure as:

$$\mu_\alpha\,\varepsilon_\alpha(t,\omega) = \varepsilon_\alpha(t,\omega) - \varepsilon^0_\alpha(t,\omega). \tag{5}$$

The second kind of the virtual distortion – force virtual distortion $p^0_k(t,\omega)$ is supposed to model modifications of inertia. Contrary to stiffness distortions $\varepsilon^0_\alpha(t,\omega)$ assigned to element α, the interia distortions correspond to k degrees of freedom of the structure. These distortions p_k^0 depend on the predetermined frequency ω of external excitation.

For a steady-state harmonic excitation, the virtual distortions can be written in the following form:

$$\varepsilon_\alpha^0(t,\omega) = \varepsilon_\alpha^0(\omega)\, \sin(\omega\, t), \qquad p_k^0(t,\omega) = p_k^0(\omega)\, \sin(\omega\, t), \tag{6}$$

where $\varepsilon_\alpha^0(\omega)$ and $p_k^0(\omega)$ are amplitudes of the generated virtual distortions.

For further considerations, let us introduce now the notion of *unit distortions*. Unit virtual distortion $\varepsilon_\alpha^0(t,\omega)$ is an initial, oscillating strain imposed on finite element that would cause strain with unit amplitude in that element taken out of structure. Analogously, force unit virtual distortion $p_k^0(t,\omega)$ is an initial, oscillating force. Onwards in all next equations, the amplitudes are used assuming the following notation:

$$\varepsilon_\alpha^0(\omega) = \varepsilon_\alpha^0, \qquad p_k^0(\omega) = p_k^0, \tag{7}$$

where ω indicates the dependency on frequency.

2.2 Influence matrices

The crucial point for VDM calculations is the influence matrix containing amplitudes obtained for unit distortions. For steady-state problems two influence matrices are generated: strain influence matrix $D_{i\beta}(\omega)$ storing displacements generated for unit strain distortions $\varepsilon_\beta^0(\omega) = 1$ and inertia influence matrix $D_{ik}(\omega)$ storing displacements generated for unit force distortions $p_k^0(\omega) = 1$.

Knowing the virtual distortions ε_α^0 and p_k^0 and influence matrices, the updated response in displacements can be calculated (without re-computing stiffness and mass matrices) as follows:

$$u_i = u_i^L + D_{i\beta}\, \varepsilon_\beta^0 + D_{ik}\, p_k^0, \tag{8}$$

where u_α^L denotes amplitudes of displacements of the original structure determined for the excitation frequency ω. Thus the actual response u_i depends on two virtual distortions $\varepsilon_\alpha^0(\omega)$ and $p_k^0(\omega)$. Multiplying Eqn (8) by $L_{\alpha Q} T_{Qi}$, the updated strain can be calculated as follows:

$$\varepsilon_\alpha = \varepsilon_\alpha^L + B_{\alpha\beta}\, \varepsilon_\beta^0 + B_{\alpha k}\, p_k^0, \tag{9}$$

where:

$$\varepsilon_\alpha = L_{\alpha Q} T_{Qi}\, u_i, \quad B_{\alpha\beta} = L_{\alpha Q} T_{Qi} D_{i\beta}, \quad B_{\alpha k} = L_{\alpha Q} T_{Qi} D_{ik}, \tag{10}$$

and $L_{\alpha Q}$ – geometry matrix, T_{Qi} – matrix of transformation to the global coordinate system.

It is necessary to quickly calculate the quantities f_A (e.g. displacement or strains), which correspond to the measured responses f_A^M. To this end, the generalized influence matrices $\breve{D}_{A\alpha}$ and $\breve{D}_{Ak}$ are built utilizing the initial matrices: $D_{i\beta}$, D_{ik},$B_{\alpha\beta}$, $B_{\alpha k}$. Finally, the updated response of a selected quantity (e.g strain) is determined in the following way:

$$f_A = f_A^L + \breve{D}_{A\alpha}\, \varepsilon_\alpha^0 + \breve{D}_{Ak}\, p_k^0, \tag{11}$$

where f_A^L denotes amplitudes of the requested responses of the original structure.

3 Problem formulation

Generally, the equations of motion for a finite element model are expressed by well-known formula:

$$\mathbf{M}\ddot{\mathbf{u}}(t) + \mathbf{C}\dot{\mathbf{u}}(t) + \mathbf{K}\mathbf{u}(t) = \mathbf{f}(t), \tag{12}$$

where $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K}$ are mass, damping and stiffness matrices, respectively and $\mathbf{f}(t)$ is the vector of external forces. In further considerations the influence of damping will be neglected. The vectors of external load and displacement for a steady-state problem are expressed as:

$$\mathbf{f}(t) = \mathbf{f}\sin(\omega t), \qquad \mathbf{u}(t) = \mathbf{u}\sin(\omega t). \tag{13}$$

Taking into account the relations in Eqn (13), the equations of motions (12) for a steady-state problem for the original structure and the modified one take the following form:

$$-\omega^2\mathbf{M}\mathbf{u} + \mathbf{K}\mathbf{u} = \mathbf{f}, \qquad \text{or} \qquad -\omega^2\mathbf{M}\mathbf{u} + \mathbf{T}\mathbf{S}\varepsilon = \mathbf{f}, \tag{14}$$

$$-\omega^2\hat{\mathbf{M}}\mathbf{u} + \hat{\mathbf{K}}\mathbf{u} = \mathbf{f}, \qquad \text{or} \qquad -\omega^2\hat{\mathbf{M}}\mathbf{u} + \mathbf{T}\hat{\mathbf{S}}\varepsilon = \mathbf{f}, \tag{15}$$

where ω – harmonic frequency, $\hat{\mathbf{M}}$ and $\hat{\mathbf{K}}$ – modified mass and stiffness matrix, $\mathbf{T}$ – transformation matrix. The matrices $\mathbf{S}$ and $\hat{\mathbf{S}}$ depend on length, axial EA and bending EJ stiffness of finite elements of the original and modified structure, respectively. Now, we can write the equations of motion (mass and stiffness matrices are intact) with imposed virtual distortions ε^0_α and p^0_k on finite elements:

$$-\omega^2\mathbf{M}\mathbf{u} + \mathbf{T}\mathbf{S}(\varepsilon - \varepsilon^{\mathbf{0}}) = \mathbf{f} + \mathbf{p}^{\mathbf{0}}. \tag{16}$$

To determine the virtual distortions p^0_k, let us compare the Eqn (15) and Eqn (16):

$$-\omega^2\,(\mathbf{M} - \hat{\mathbf{M}})\mathbf{u} = \mathbf{p}^0. \tag{17}$$

The difference $M_{ij} - \hat{M}_{ij}$ for beam structures can be expressed in the following way:

$$M_{ij} - \hat{M}_{ij} = \Delta M_{ij} = (1 - \mu^A_\gamma)\overset{\mathrm{A}}{M}{}^\gamma_{ij} + (1 - \mu^J_\gamma)\overset{\mathrm{J}}{M}{}^\gamma_{ij}, \tag{18}$$

where $\mu^A_\gamma = \frac{\hat{A}_\gamma}{A_\gamma}$, $\mu^J_\gamma = \frac{\hat{J}_\gamma}{J_\gamma}$ are modification parameters for element γ. The mass matrix is decomposed into matrix $\overset{\mathrm{A}}{M}{}^\gamma_{ij}$ – depending on cross-section area A_γ and matrix $\overset{\mathrm{J}}{M}{}^\gamma_{ij}$ – depending on moment of interia J_γ. Let us notice that the following relation holds:

$$M_{ij} = \sum_\gamma \overset{\mathrm{A}}{M}{}^\gamma_{ij} + \sum_\gamma \overset{\mathrm{J}}{M}{}^\gamma_{ij}. \tag{19}$$

In order to determine the virtual distortions ε^0_α, p^0_k, let us substitute Eqn (9) to Eqn (5) and Eqn (8) to Eqn (17), yielding:

$$\begin{bmatrix} \delta_{\alpha\beta} - (1-\mu_\alpha)\,B_{\alpha\beta} & -(1-\mu_\alpha)\,B_{\alpha k} \\ -\omega^2\Delta M_{ij}D_{j\beta} & \delta_{ik} - \omega^2\Delta M_{ij}D_{jk} \end{bmatrix} \begin{bmatrix} \varepsilon^0_\beta \\ p^0_k \end{bmatrix} = \begin{bmatrix} (1-\mu_\alpha)\,\varepsilon^L_\alpha \\ \omega^2\Delta M_{ij}u^L_j \end{bmatrix}. \tag{20}$$

The calculated virtual distortions ε^0_α, p^0_k (for assumed vector of stiffness parameters μ_α) from the set of equation (20) are used to compute updated response f_A corresponding to the measured one f^M_A. Further, the vector of stiffness modification parameters μ_α is iteratively determined by minimisation of the proposed objective function:

$$F(\mu_\alpha) = \sum_\omega \sum_A \left(f_A - f^M_A\right)^2, \tag{21}$$

using the gradient approach.

4 Damage identification technique

In order to determine the control parameters μ_α, an iterative update of the modification parameters is proposed:

$$\mu_\alpha^{(i+1)} = \mu_\alpha^{(i)} - \Delta F^{(i)} \frac{\nabla F^{(i)}}{\nabla F^{(i)} \left[\nabla F^{(i)}\right]^T}, \tag{22}$$

where

$$\nabla F^{(i)} = \frac{\partial F^{(i)}}{\partial \mu_\alpha^{(i)}} = \begin{bmatrix} \frac{\partial F^{(i)}}{\partial \varepsilon_\beta^{(i)0}} \frac{\partial \varepsilon_\beta^{(i)0}}{\partial \mu_\alpha^{(i)}} \\ \frac{\partial F^{(i)}}{\partial p_k^{(i)0}} \frac{\partial p_k^{(i)0}}{\partial \mu_\alpha^{(i)}} \end{bmatrix} = 2 \sum_\omega \sum_A \left(f_A - f_A^M\right) \begin{bmatrix} \breve{D}_{A\beta} \frac{\partial \varepsilon_\beta^{(i)0}}{\partial \mu_\alpha^{(i)}} \\ \breve{D}_{Ak} \frac{\partial p_k^{(i)0}}{\partial \mu_\alpha^{(i)}} \end{bmatrix}, \tag{23}$$

is the gradient of the objective function in $i-th$ iteration. For reaching the optimum solution of the function (21), the gradients $\frac{\partial \varepsilon_\beta^0}{\partial \mu_\alpha}$ and $\frac{\partial p_k^0}{\partial \mu_\alpha}$ have to be calculated. To this end, let us differentiate the Eqn (20) with respect to modification parameters μ_α:

$$\begin{bmatrix} \delta_{\alpha\beta} - (1-\mu_\alpha) B_{\alpha\beta} & -(1-\mu_\alpha) B_{\alpha k} \\ -\omega^2 \Delta M_{ij} D_{j\beta} & \delta_{ik} - \omega^2 \Delta M_{ij} Djk \end{bmatrix} \begin{bmatrix} \frac{\partial \varepsilon_\beta^0}{\partial \mu_\alpha} \\ \frac{\partial p_k^0}{\partial \mu_\alpha} \end{bmatrix} = \begin{bmatrix} -\varepsilon_\alpha \\ \omega^2 \frac{\partial \Delta M_{ij}}{\partial \mu_\alpha} u_j \end{bmatrix}. \tag{24}$$

Let us notice in Eqn (24) that the left-hand side matrix is the same as in Eqn (20), whereas the right-hand side depends on updated displacements and strains.

5 Numerical examples

5.1 Cantilever beam

As an illustration of the discussed damage identification method, let us consider a simple cantilever beam divided into 25 finite elements, shown in Fig. (2). The original parameters are identical in all finite elements:

- cross-section area $A = 1 \cdot 10^{-4}\, m^2$,
- moment of interia $J = 1.0417 \cdot 10^{-12}\, m^4$.

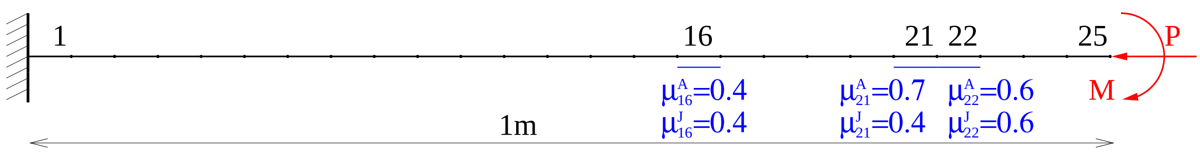

Figure 2: Tested 2D beam structure.

The harmonic excitation – bending moment and axial force – is applied to the free end of the cantilever and expressed by the relations:

$$M = M^0 \sin(\omega t), \qquad P = P^0 \sin(\omega t), \tag{25}$$

with the amplitudes: $M^0 = 1\,[Nm]$, $P^0 = 100\,[kN]$ and the applied frequency $\omega = \{50, 100, 220\}$ $[Hz]$ (out of resonance). The measured data were numerically simulated for each frequency ω (all components of strain responses ε_α^M). The results of inverse analysis are presented in Fig. (3) for cross-section modification μ_α^A, and in Fig. (4) – for the moment of inertia modification μ_α^J.

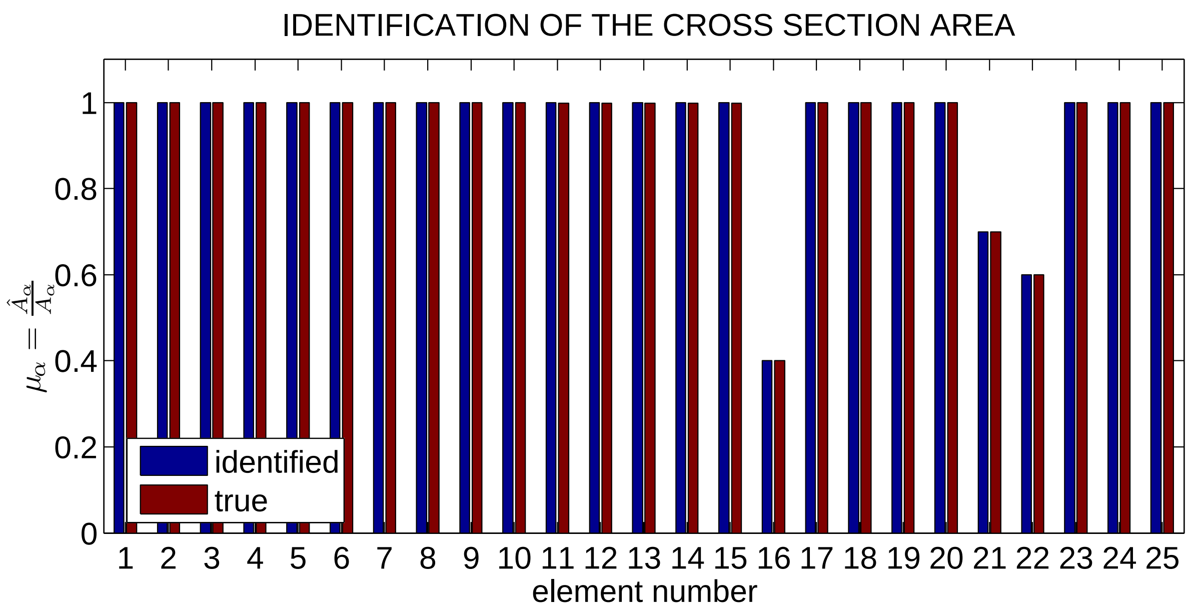

Figure 3: Identified cross-section areas after 500 iterations.

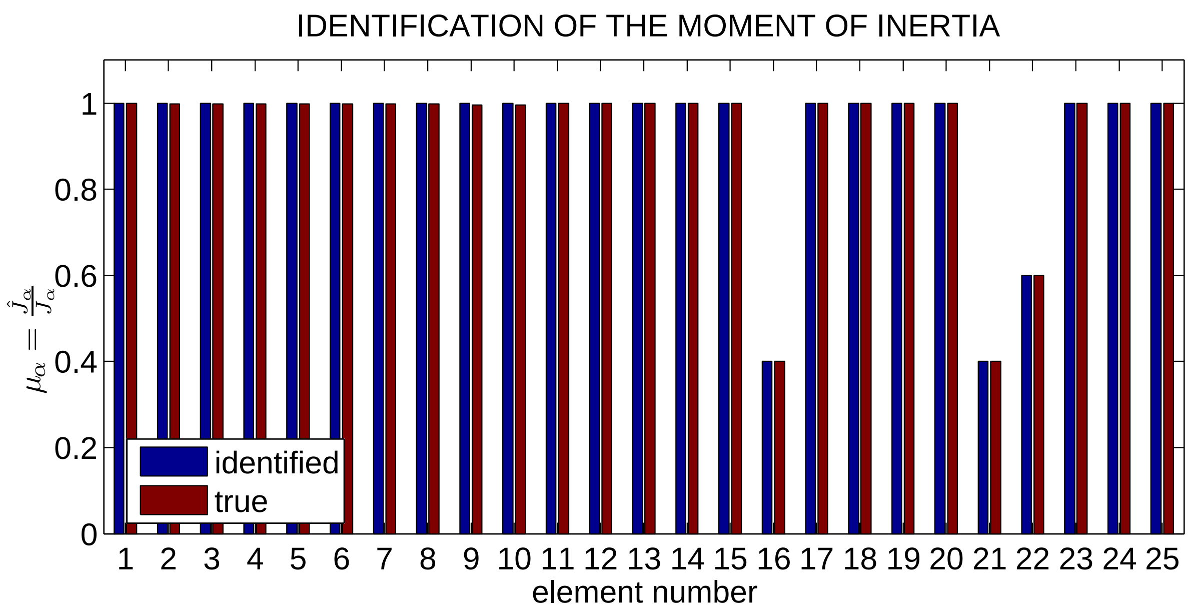

Figure 4: Identified moments of inertia after 500 iterations.

5.2 2D Truss structure

For the truss finite element, there is only one variable to be modelled, namely cross-section area modification $\mu^A_\alpha = \frac{\hat{A}}{A}$. Thus for the structure presented in Fig. (5), there are 20 unknown parameters μ_α. In Fig. (6), the results of the frequency-domain approach (VDM-F) are compered with the outcome of the previously described (see [2]) time-domain approach (VDM - time domain).

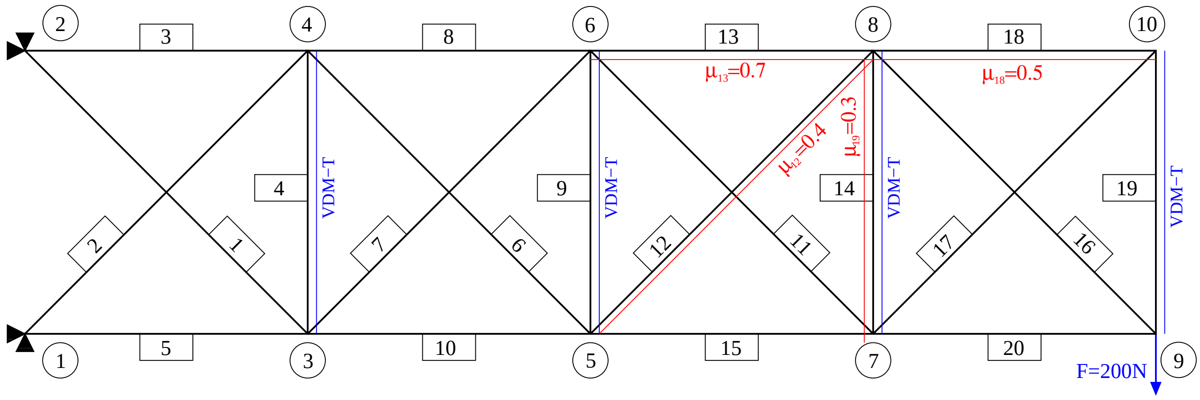

Figure 5: Tested 2D truss structure

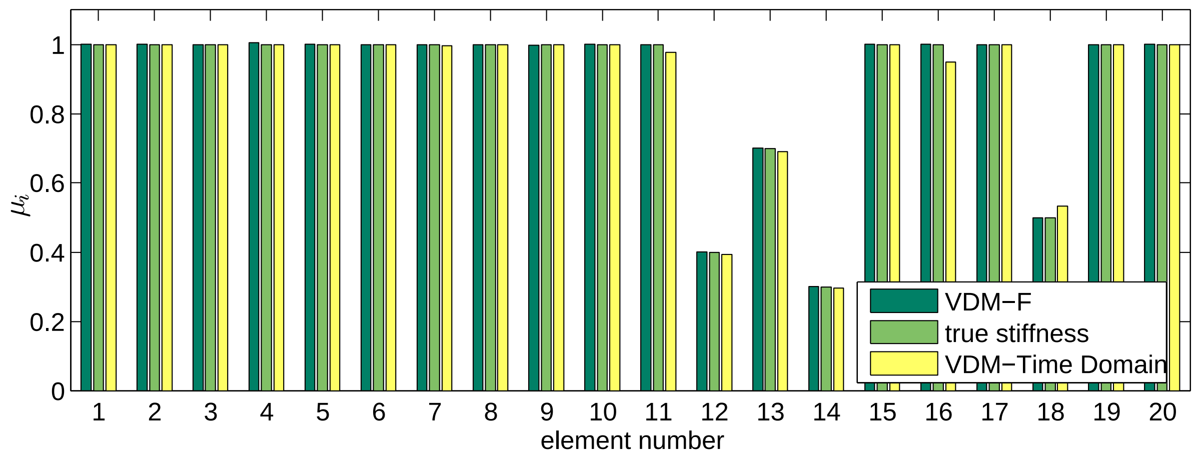

Figure 6: Numerical results of the tested 2D truss structure

6 Experimental stand

The experimental stand of a simply supported 3D steel truss structure (70 elements) is presented in Fig. (7). The harmonic excitation is realised by a piezo-actuator in the middle of the structure (see lower left corner in Fig. (7b)). The response of the structure is measured by thin piezo-patch sensors glued on elements and transmitted to oscilloscope. In the truss demonstrator, various damage scenarios due to replacing the initial truss elements with other ones of different stiffness and mass will be investigated. The experiment is now at the stage of matching model parameters to experimental responses.

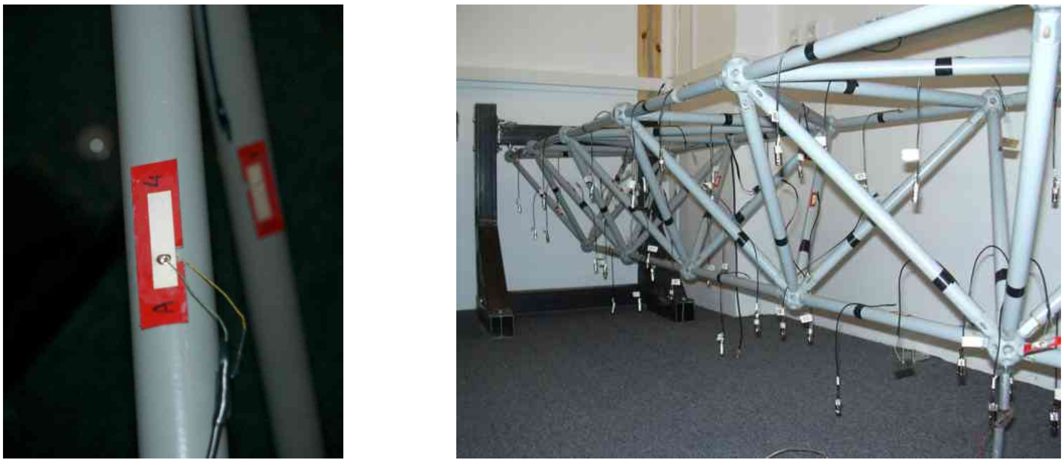

Figure 7: Experimental stand – 3D truss structure. (a) piezo-sensor, (b) general view.

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